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In[1]:= Q[f_, M_] := Block[{θ1, θ2, c1, c2, M21, w21},
  θ1 = f * θ2; M21 = M;
  c1 = 2 / θ1; c2 = 2 / θ2; w21 = 4 M21 / θ1;

  {{-(2 * w21 + c1), w21, w21, 0, c1, 0, 0, 0, 0, 0},
   {0, -(w21 + c2), 0, w21, 0, c2, 0, 0, 0, 0},
   {0, 0, -(w21 + c2), w21, 0, 0, c2, 0, 0, 0},
   {0, 0, 0, -3 * c2, 0, 0, 0, c2, c2, c2},
   {0, 0, 0, 0, -w21, 0, 0, w21, 0, 0},
   {0, 0, 0, 0, 0, -w21, 0, 0, w21, 0},
   {0, 0, 0, 0, 0, 0, -c2, 0, 0, c2},
   {0, 0, 0, 0, 0, 0, 0, -c2, 0, c2},
   {0, 0, 0, 0, 0, 0, 0, 0, -c2, c2},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
];

PG1a[f_, M_] := Block[{Esys, U, Evalues, P, θ1, θ2, τ},
  θ1 = f * θ2;
  Esys = Eigensystem[Q[f, M]];
  Evalues = Esys[[1]];
  If[Abs[Evalues[[11]]] > 10^(-20), Print["eigenvalue 11 is not 0?"]];
  Evalues[[11]] = -1; (* eigenvalues are ordered increasingly, last one is 0. *)
  Evalues = (Exp[Evalues * τ] - 1) / Evalues;
  Evalues[[11]] = τ;
  U = Transpose[Esys[[2]]];
  P = U . DiagonalMatrix[Evalues] . Inverse[U];
  P[[1, 1]] * 2 / θ1 + P[[1, 4]] * 2 / θ2
];

PG1b[f_, M_] := Block[{P, τ},
  P = MatrixExp[Q[f, M] * τ];
  (P[[1, 1]] + P[[1, 2]] + P[[1, 3]] + P[[1, 4]]) / 3
];

PG1[f_, M_] := PG1a[f, M] + PG1b[f, M];
gdiJ[f_, M_] := (PG1[f, M] - 1 / 3) * (3 / 2);

θ2 = 0.001; τ = 0.005; f = 5.0; M = 1;

Plot[{gdiJ[f, M], PG1a[f, M], 0}, {f, 0.1, 5}, Frame → {{True, False}, {True, False}},
  FrameStyle → Directive[AbsoluteThickness[Medium], Black, 12],
  FrameTicks → {{{0, 0, .025}, {1, 1, .025}, {2, 2, .025}, {3, 3, .025},
    {4, 4, .025}, {5, 5, .025}}, {{-0.2, -0.2, .025}, {0, 0, .025},
    {0.2, 0.2, .025}, {0.4, 0.4, .025}, {0.6, 0.6, .025}, {0.8, 0.8, .025}}},
  PlotRange → {All, {-0.2, 0.9}}, AxesOrigin → {0.1, -0.2}, AspectRatio → .8]
Plot[{gdiJ[f, M], PG1a[f, M], 0}, {M, 0, 2}, Frame → {{True, False}, {True, False}},
  FrameStyle → Directive[AbsoluteThickness[Medium], Black, 14],
  FrameTicks → {{{0, 0, .025}, {0.5, 0.5, .025}, {1, 1, .025},
    {1.5, 1.5, .025}, {2, 2, .025}}, {{-0.2, " ", .025}, {0, "", .025},
    {0.2, " ", .025}, {0.4, " ", .025}, {0.6, " ", .025}, {0.8, " ", .025}}},
  PlotRange → {All, {-0.2, 0.9}}, AxesOrigin → {0, -0.2}, AspectRatio → .8]

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