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In[6]:= (* P(G1) and P(G1a), under the IM model for 3 sequences (a1, a2, b) for heuristic species delimitation.
The notation is from Kornai et al. (2024).
PG1a works if M>0 only. PG1aInt works if M is specified with high precision,
such as M = 10`100.
*)
02 = 0.001;  $\tau$  = 0.005;
Q[f_, M_] := Block[{01, 02, c1, c2, M21, w21},
  01 = f * 02; M21 = M;
  c1 = 2 / 01; c2 = 2 / 02; w21 = 4 M21 / 01;

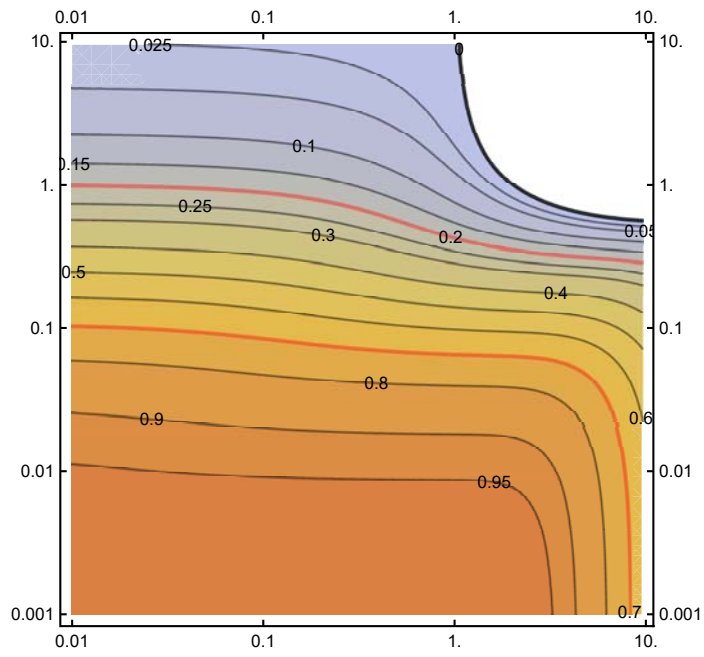
  {{-(2 * w21 + c1), w21, w21, 0, c1, 0, 0, 0, 0, 0},
   {0, -(w21 + c2), 0, w21, 0, c2, 0, 0, 0, 0},
   {0, 0, -(w21 + c2), w21, 0, 0, c2, 0, 0, 0},
   {0, 0, 0, -3 * c2, 0, 0, 0, c2, c2, c2},
   {0, 0, 0, 0, -w21, 0, 0, w21, 0, 0},
   {0, 0, 0, 0, 0, -w21, 0, 0, w21, 0},
   {0, 0, 0, 0, 0, 0, -c2, 0, 0, c2},
   {0, 0, 0, 0, 0, 0, 0, -c2, 0, c2},
   {0, 0, 0, 0, 0, 0, 0, 0, -c2, c2},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
];
PG1a[f_, M_] := Block[{Esys, U, Evalues, P, 01, 02,  $\tau$ },
  01 = f * 02;
  Esys = Eigensystem[Q[f, M]];
  Evalues = Esys[[1]];
  If[Abs[Evalues[[11]]] > 10^(-20), Print["eigenvalue 11 is not 0?"]];
  Evalues[[11]] = -1; (* eigenvalues are ordered increasingly, last one is 0. *)
  Evalues = (Exp[Evalues *  $\tau$ ] - 1) / Evalues;
  Evalues[[11]] =  $\tau$ ;
  U = Transpose[Esys[[2]]];
  P = U . DiagonalMatrix[Evalues] . Inverse[U];
  P[[1, 1]] * 2 / 01 + P[[1, 4]] * 2 / 02
];
PG1b[f_, M_] := Block[{P,  $\tau$ },
  P = MatrixExp[Q[f, M] *  $\tau$ ];
  (P[[1, 1]] + P[[1, 2]] + P[[1, 3]] + P[[1, 4]]) / 3
];
PG1[f_, M_] := PG1a[f, M] + PG1b[f, M];

In[11]:= b = 10; Mmin = 0.001; Mmax = 9.5; fmin = 0.01; fmax = 9.5;
newStyle[x_] := x /. l_Line -> Sequence[Opacity[.4], Thick, Red, 1]
newStyle2[x_] := x /. l_Line -> Sequence[Opacity[.8], Thick, Black, 1]
ContourPlot[(PG1[b^f, b^M] - 1/3) * 3/2,
  {f, Log[b, fmin], Log[b, fmax]}, {M, Log[b, Mmin], Log[b, Mmax]},
  Contours -> {0, 0.025, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95},
  (* ContourStyle -> {{Red, Thin}}, *)
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 9}, PlotPoints -> 30, ContourLabels -> All,
  ColorFunction -> (ColorData[{"BeachColors", "Reverse"}]), ContourStyle -> Thin, AspectRatio -> 1,
  (* Frame -> False, *)
  FrameTicks -> {Table[{f, ToString[Round[b^f, fmin]]}, {f, Log[b, fmin], Log[b, 100]}],
    Table[{M, ToString[Round[b^M, Mmin]]}, {M, Log[b, Mmin], Log[b, 50]}]}
] /. Tooltip[x_, 0.2] -> Tooltip[newStyle[x], 0.2] /.
Tooltip[x_, 0.7] -> Tooltip[newStyle[x], 0.7] /. Tooltip[x_, 0] -> Tooltip[newStyle2[x], 0]

ContourPlot[PG1a[b^f, b^M], {f, Log[b, fmin], Log[b, fmax]}, {M, Log[b, Mmin], Log[b, Mmax]},
  Contours -> {0, 0.025, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95},
  BaseStyle -> {FontFamily -> "Arial", FontSize -> 9}, PlotPoints -> 30, ContourLabels -> All,
  ColorFunction -> (ColorData[{"BeachColors", "Reverse"}]), ContourStyle -> Thin, AspectRatio -> 1,
  (* Frame -> False, *)
  FrameTicks -> {Table[{f, ToString[Round[b^f, fmin]]}, {f, Log[b, fmin], Log[b, 50]}],
    Table[{M, ToString[Round[b^M, Mmin]]}, {M, Log[b, Mmin], Log[b, 10]}]}
] /. Tooltip[x_, 0.2] -> Tooltip[newStyle[x], 0.2] /.
Tooltip[x_, 0.7] -> Tooltip[newStyle[x], 0.7]

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Out[14]=



Out[15]=

